Transverse Mode Properties of Beam-Rotated Unstable Resonators for Free Electron Lasers

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A novel class of ring unstable resonators (UR90) with 90-deg rotation of the beam on each round-trip has been proposed as a method of avoiding disadvantages of obscuration in the output beam. We herein consider extension of this idea to include an internally focused beam, so that the portion of the beam near the focus can fit within the narrow gain region of a free electron laser. Modeling of transverse mode properties of empty cavities with or without an internal focus proceeds in the same general way. We have calculated the transverse mode intensity and phase distributions and the power retention fraction *PR* as a function of equivalent Fresnel number F_{eq} over various ranges of F_{eq}. There is quasi-oscillatory behavior similar to that found in conventional unstable resonators, with poor mode loss separation occurring near an interger *N* plus 7/8; hence, designs should avoid such values of F_{eq}.

Introduction

FREE electron lasers (FEL's) operated as high-power oscillators present special problems regarding resonator design. The optical beam must be small in transverse extent in the (wiggler) gain region, but the optical flux on resonator mirrors must not be allowed to damage them. The flux can be reduced to acceptable levels in at least two ways. Very long paths from gain region to mirrors can be used to spread the beam by diffraction, but the great distances involved are undesirable. Alternatively, grazing incidence beam expanders/contractors can be used. With the latter, one has the choice of operation in either the stable or the unstable domain. Stable resonators can provide the advantage of an unobscured output beam if a practical means is available to separate out a fixed fraction of the total circulating beam for output, but this poses practical problems (materials damage, etc.) at high flux levels. Operation in the unstable regime avoids the latter difficulty by using a scraper output mirror that outputs all of the outer portion of the beam, i.e., the scraper need not be a partially transmitting element. The special resonator problems of FEL's are, of course, related to the general question of choice of most favorable resonator design, e.g., stable or unstable; positive or negative branch; conventional or, perhaps, novel designs for special applications, etc.

The excellent transverse mode separation properties of (both positive and negative branches) unstable resonators are well known and such resonators are very widely used. Because of the internal focus of the negative branch type and the possibility of breakdown in gaseous or solid gain media, the positive branch class is most widely used. For FEL's the breakdown problem is absent because gain takes place in a vacuum, and negative branch resonators are acceptable.

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A major disadvantage of unstable resonators is the presence of an obscuration in the output beam that is inherent in the general design. This is most troublesome for cases with centered alignment, which is the most common configuration. The obscuration results in broadening of the focused spot of the beam and, indeed, in removal of a substantial fraction of the power from the main lobe. For low-gain systems (such as some FEL's) with a correspondingly large obscuration fraction, the far-field deterioration can be quite serious and possibly unacceptable. If one could devise a resonator configuration that combines the good mode-loss-separation properties of conventional unstable resonators with the good focused spot properties of an unobscured output beam, it would be very desirable.

A general resonator configuration that may combine the desirable features noted above has, in fact, been proposed. It consists of an arrangement in which the recirculating resonator beam is rotated about the propagation axis by a specified angle on each pass. The particular design of interest here involves a rotation by 90 deg on each pass; for brevity it is referred to as a UR90 resonator. It has been shown that the transverse mode properties of UR90 are equivalent to those of a negative branch unstable strip resonator that is conventional except that the output is from a one-sided scraper mirror. This equivalence makes it feasible to calculate the mode properties by use of computer programs that are generally similar to those for conventional resonators but are modified in regard to the single-sided scraper. Whether or not the rotated-beam design does actually have the desirable mode-loss separation properties of conventional unstable resonators can, of course, only be determined by calculations or experiments. In this paper, we present results of calculations of transverse modes of such resonators for a range of values of equivalent Fresnel number and for a few selected values of resonator magnification.

The earliest proposal for beam-rotation resonators seems to have come from the Russians. Indeed, Kuprenyuk et al.[1] calculated the transverse mode properties of a UR90 resonator for a small range of resonator parameters. Paxton and Latham[2] have elaborated significantly on the UR90 design and have explicitly demonstrated the equivalence of such a resonator to a negative branch, one-sided scraper resonator with parameters related in a certain way to those of the actual UR90 device. We have adopted the equivalence relations as given by them[2] in our calculations. As a proof test of our extended computer program, we have satisfactorily reproduced the intensity and phase distribution of a particualr design that was treated by Paxton and Latham.[2]

Paxton[3] has shown that the modes and eigenvalues of a resonator with a negative equivalent Fresnel number are the complex conjugates of those of the resonator with the same magnification and a positive equivalent Fresnel number of the same magnitude. For simplicity in the present discussions, we consider two modes to be essentially equivalent if the magnitude of the eigenvalue, the transverse variation of intensity, and the transverse variation of relative phase without regard to sense of the variation are the same. Thus we consider as essentially equivalent the modes of two resonators that differ only in the sign of the equivalent Fresnel number. We note also that the sense of the transverse variation of phase depends on whether one uses the propagation convention of exp $(+ikz)$ or of exp $(-ikz)$.

Description of UR90 Resonators

To our knowledge, previous specifically proposed UR90 resonators are analogous to conventional resonators used with gaseous or solid gain media in that the circulating beam does not pass through a focus within the resonator. We wish to consider an alternate design in which the beam does pass through an internal focus, thereby producing a localized narrow beam suitable for amplification within the wiggler of a free electron laser. (Such possible use of a negative branch afocal telescope in the beam expanding portion of a UR90 was already mentioned briefly by Paxton and Latham.[2]) These two generic types of UR90 resonators, without and with an internal focus, are depicted in Figs. 1 and 2.

Figure 1 shows a UR90 unstable ring resonator of internally unfocused beam type. For simplicity the optical elements that expand and then recollimate the beam are depicted as lenses, although in practice they will more likely be mirrors. The design is rather conventional except for two features First, there is an additional optical element labeled "90 deg rotator" that rotates the beam by 90 deg about the propagation axis. For discussions of how this rotation can be physically accomplished, one may refer to earlier papers.[1,2] Second, the scraper mirror is one-sided, i.e., it reflects into the output beam only radiation falling on one half plane. By contrast, the scraper of a conventional resonator (without a beam rotator) fills a full plane except for the rectangular opening that transmits the inner portion of the beam for recirculation through the resonator.

Figure 2 shows a UR90 unstable ring resonator of internally focused beam type, the variant that we propose for use with free electron lasers. This resonator has a focus within the gain region (analogous to a conventional negative branch resonator). It turns out that the analysis and computations for the types of resonators in Figs. 1 and 2 are very similar.

A cross section of the circulating beam just ahead of the scraper mirror is shown in Fig. 3. For brevity, the description here may be somewhat incomplete. Consider the overall rectangular beam defined by points *a, b, c, d, e, f, a*. The portion of the beam designed by points *c, d, e, f, c* strikes the scraper mirror (indicated as being within the region surrounded by dashes) and is directed into the output beam. The portion *a, b, c, f, a* passes through the resonator again. During its passage, it is expanded in size and rotated by 90 deg. The clockwise or counterclockwise sense of rotation can be selected by choice of the physical elements that rotate the beam. Figure 3 as drawn refers to a design with clockwise beam rotation. Note that a design with an internal focus also will invert the beam, so that the combined effect of beam rotation and inversion is simply a beam rotation in the opposite sense. A detailed discussion of the relative dimensions, etc., in Fig. 3 can be found in Ref. 2.

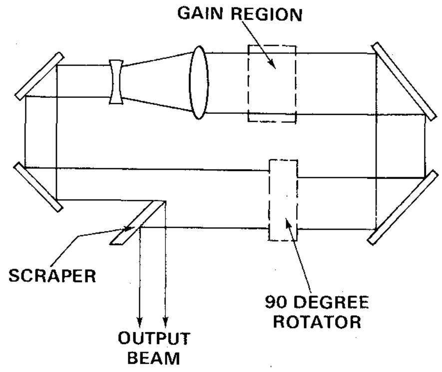

Fig. 1 UR90 unstable ring resonator of internally unfocused beam type.

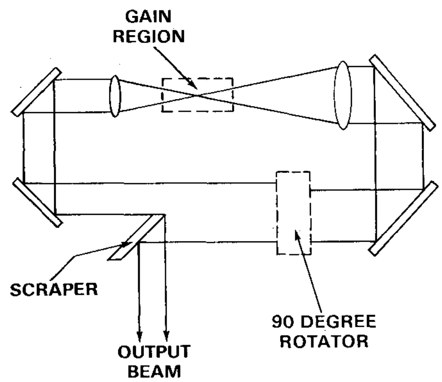

Fig. 2 UR90 unstable ring resonator of internally focused beam type.

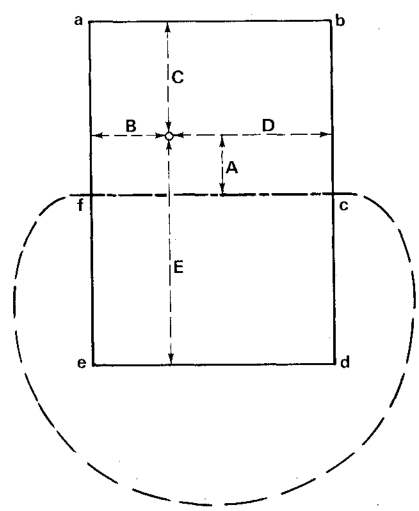

Fig. 3 Cross section of the circulating beam of a UR90 resonator just ahead of the scraper mirror. The optic axis is indicated by the small circle. The beam undergoes a 90-deg clockwise rotation on each resonator pass.

The functioning of a UR90 differs from that of a conventional unstable resonator in the crucial respect that the (rectangular) output beam is totally unobscured. Therefore, the far-field properties are much better, especially for cases with low magnification.

It is perhaps physically clear that after two successive passages through the ring resonator, the optical beam will be inverted. Hence, two successive passes in the physical resonator are essentially equivalent to a single pass through a conventional negative branch resonator with a one-sided feedback mirror. The mathematics have been worked out by Pax-

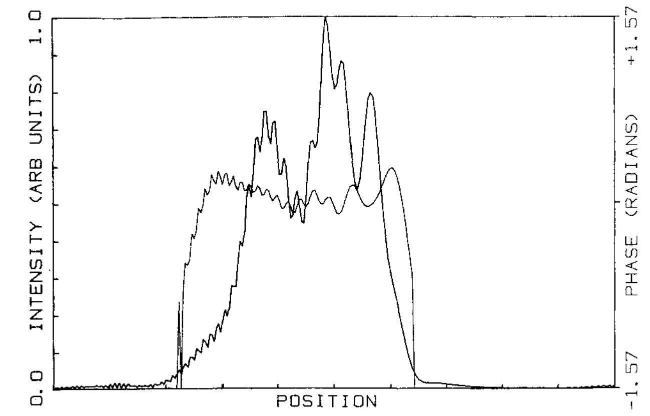

Fig. 4 Calculated intensity and phase distribution for a UR90 resonator with $M=-1.86$, $F_{eq}=-1.3437$; the curve with higher peaks represents intensity.

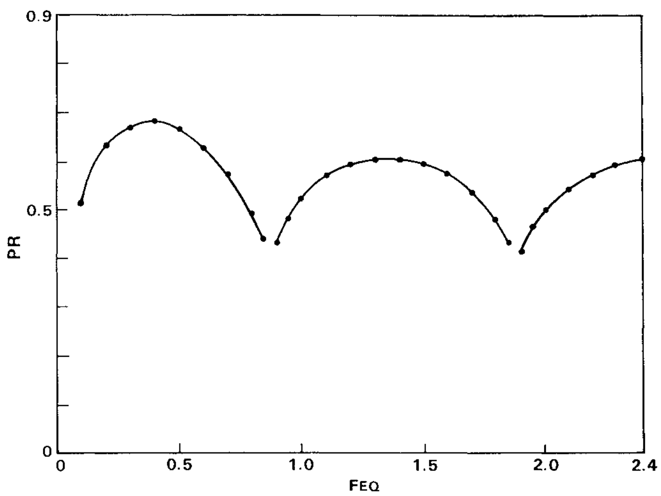

Fig. 5 Calculated values of power retention ratio *PR* as a function of F_{eq} in the range of 0 to 2.4 for $M=-1.86$.

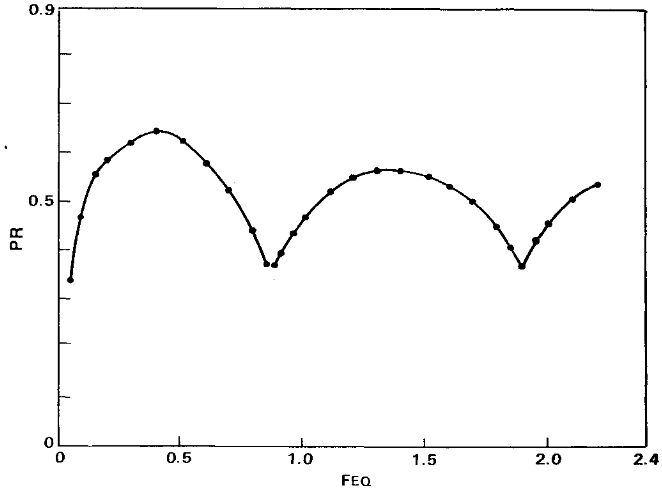

Fig. 6 Calculated values of power retention ratio *PR* as a function of F_{eq} in the range of 0 to 2.4 for $M=-2.0$.

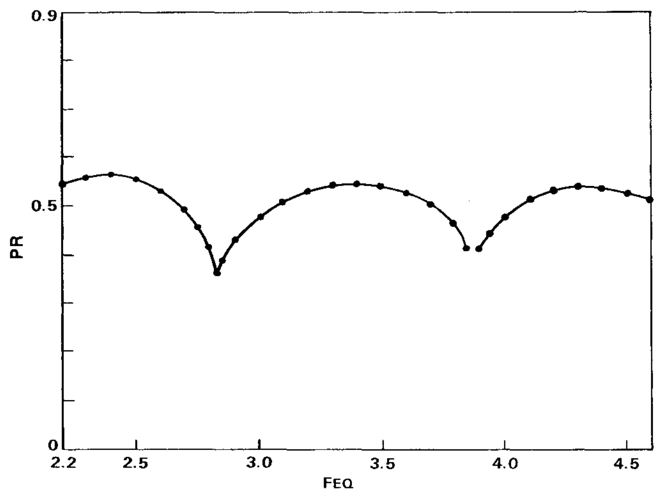

Fig. 7 Calculated values of power retention ratio *PR* as a function of F_{eq} in the range of 2.2 to 4.6 for $M=-2.0$.

ton and Latham.[2] We have carried out calculations by two distinct schemes and confirmed that the results are essentially the same.

As is customary in calculations of empty resonator properties, we neglect effects of gain or of effective aperturing resulting from real-world details of device structure, etc. Such neglect is probably more significant for FEL's than for more conventional lasers, particularly because of the inherent tendency for the gain to be transversely nonuniform (cylindrically symmetric). On the other hand, the present results are not limited to FEL devices, even though they have been obtained with such devices primarily in mind. They are applicable in the strictly empty cavity approximation for both positive and negative ranch unstable resonators with 90-deg beam rotation.

Results

Confirmation of proper functioning of our computer programs CAVTOS and CSOSC1 was obtained first by using each of them to treat a case with $M=-1.86$, $F_{eq}=-1.3437$, which was also treated by Paxton and Latham.[2] Plots of the intensity and phase distribution obtained with the CAVTOS program, which treats the propagation by a Fourier transform method, are given in Fig. 4. These are in good agreement with the results of Paxton and Latham.[2] The relatively small variations in phase of the beam indicate that it should have rather good optical quality as regards propagation to a focal point at a target. We calculated this case using various combinations of mesh point numbers NTOT and NMIR, with results that indicate no critical dependence on choice of mesh point size. Also, we treated this case with the program CVOSC1, which calculates propagation by a convolution theorem evaluation of the Fresnel-Kirchoff integral, and the results were very nearly the same. In addition, some other combinations of M and F_{eq} were treated with both CAVTOS and CVOSC1; agreement between results from the two methods was satisfactory in each case.

A series of calculations was carried out for various values of F_{eq}, with M held fixed at -1.86. Resulting values of power retention ratio *PR* ($PR=1-$outcoupling fraction) are plotted in Fig. 5. Similar plots have often been presented to summarize results of transverse mode calculations of conventional unstable resonators with two-sided scrapers.[4-6] For the conventional systems with this value of magnification, there are sharp cusps or crossings at values of F_{eq} near $N+7/8$, where N is zero or a positive integer.[4-6] A qualitatively similar behavior is evidenced in Fig. 5.

We have chosen to concentrate further attention in the currently reported work on cases with $M=-2.0$. (This choice was related to our earlier calculations of properties of conventional resonators.[5,6]) In particular, we have calculated and plotted the power retention ratio as a function of F_{eq}, as given in Figs. 6-9. One sees immediately that there is a quasiperiodicity of *PR* as a function of F_{eq}, rather much as for

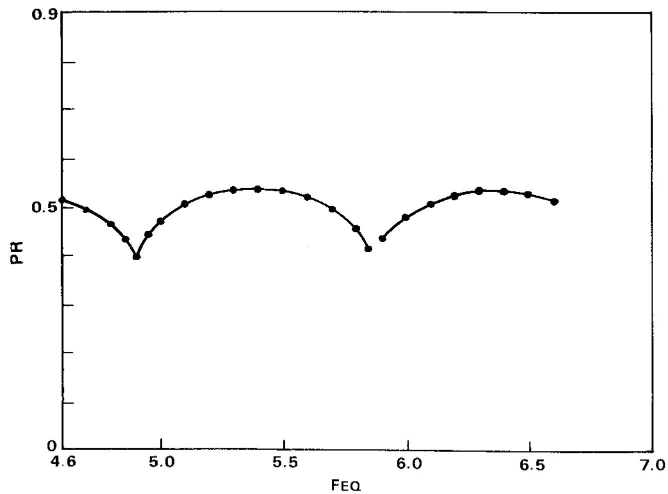

Fig. 8 Calculated values of power retention ratio *PR* as a function of F_{eq} in the range of 4.6 to 7.0 for $M = -2.0$.

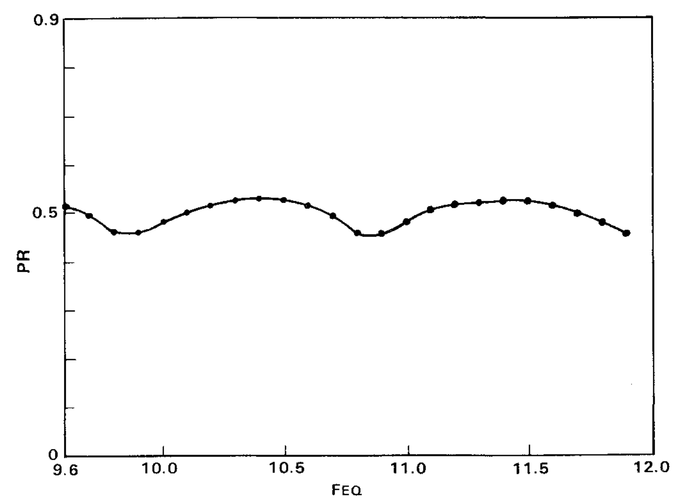

Fig. 9 Calculated values of power retention ratio *PR* as a function of F_{eq} in the range of 9.6 to 12.0 for $M = -2.0$.

conventional unstable resonators.[4-6] This has been confirmed here for $M = -2.0$ and F_{eq} ranging from 0 to 9.6.

We have not directly calculated the mode loss of any but the fundamental mode. On the basis of past experience, however, one confidently expects that the mode loss separation properties will be good in cases where the calculated value of *PR* for the fundamental mode is greater than the nominal, geometric optics, value, and convergence of the calculations is relatively rapid, just as for conventional unstable resonators. The mode loss separation properties will be poor in cases where the calculated value of *PR* for the fundamental mode is less than the nominal, geometric optics, value, and convergence of the calculations is relatively slow. (By rapid convergence, we mean that relatively few iterations of the calculation of propagation of radiation around the resonator are required to obain a self-consistent solution.) The present calculations are consistent, in that good convergence occurs whenever *PR* is greater than nominal, and poor convergence occurs whenever *PR* is less than nominal. Thus, it is thought that mode loss separation will be poor for cases with F_{eq} near $N+7/8$ but will be good for intermediate values of F_{eq}. This is quite similar to the case for conventional positive-branch unstable resonators except that, for the latter, small values of misalignment can also lead to poor mode-loss-separation for F_{eq} values near $N+3/8$,[5,6] whereas no such possibility arises for UR90 resonators.

Acknowledgment

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